EXERCICES DE

CALCUL VECTORIEL

EXERCICE 1.

Niveau : Université
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Mots-clés :

Enoncé :

Prouvez que :

$$rot[\overrightarrow{grad}(f)] = \vec{0}$$

Solution :

$$rot[\overrightarrow{grad}(f)] = \vec{\nabla} \times (\vec{\nabla} f) = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \dfrac{\partial f}{\partial x} & \dfrac{\partial f}{\partial y} & \dfrac{\partial f}{\partial z} \end{vmatrix}$$

$$= \frac{\partial^2 f}{\partial y \partial z} \cdot \vec{e}_1 - \frac{\partial^2 f}{\partial y \partial z} \cdot \vec{e}_1 - \frac{\partial^2 f}{\partial x \partial z} \cdot \vec{e}_2 + \frac{\partial^2 f}{\partial z \partial x} \cdot \vec{e}_2 + \frac{\partial^2 f}{\partial x \partial y} \cdot \vec{e}_3 - \frac{\partial^2 f}{\partial y \partial x} \cdot \vec{e}_3$$

$$= \left[\frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y} \right] \cdot \vec{e}_1 + \left[\frac{\partial^2 f}{\partial z \partial x} - \frac{\partial^2 f}{\partial x \partial z} \right] \cdot \vec{e}_2 + \left[\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial y \partial x} \right] \cdot \vec{e}_3$$

$$= 0 \cdot \vec{e}_1 + 0 \cdot \vec{e}_2 + 0 \cdot \vec{e}_3 = \vec{0}$$

C.Q.F.D.

EXERCICE 2.

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Mots-clés :

Enoncé :

Montrez que le gradient est une application linéaire.

Solution :

Soit f et g deux fonctions ;

$\overrightarrow{grad}(\alpha\cdot f+\beta\cdot g)=\vec{\nabla}(\alpha\cdot f+\beta\cdot g)$ où α et β sont des constantes

$$=\frac{\partial}{\partial x}(\alpha\cdot f+\beta\cdot g)\cdot\vec{e}_1+\frac{\partial}{\partial y}(\alpha\cdot f+\beta\cdot g)\cdot\vec{e}_2+\frac{\partial}{\partial z}(\alpha\cdot f+\beta\cdot g)\cdot\vec{e}_3$$
$$=\frac{\partial}{\partial x}(\alpha\cdot f)\cdot\vec{e}_1+\frac{\partial}{\partial x}(\beta\cdot g)\cdot\vec{e}_1+\frac{\partial}{\partial y}(\alpha\cdot f)\cdot\vec{e}_2+\frac{\partial}{\partial y}(\beta\cdot g)\cdot\vec{e}_2+\frac{\partial}{\partial z}(\alpha\cdot f)\vec{e}_3+\frac{\partial}{\partial z}(\beta\cdot g)\vec{e}_3$$
$$=\alpha\cdot\frac{\partial f}{\partial x}\cdot\vec{e}_1+\alpha\cdot\frac{\partial f}{\partial y}\cdot\vec{e}_2+\alpha\cdot\frac{\partial f}{\partial z}\cdot\vec{e}_3+\beta\cdot\frac{\partial g}{\partial x}\cdot\vec{e}_1+\beta\cdot\frac{\partial g}{\partial y}\cdot\vec{e}_2+\beta\cdot\frac{\partial g}{\partial z}\cdot\vec{e}_3$$
$$=\alpha\cdot\left(\frac{\partial f}{\partial x}\cdot\vec{e}_1+\frac{\partial f}{\partial y}\cdot\vec{e}_2+\frac{\partial f}{\partial z}\cdot\vec{e}_3\right)+\beta\cdot\left(\frac{\partial f}{\partial x}\cdot\vec{e}_1+\frac{\partial f}{\partial y}\cdot\vec{e}_2+\frac{\partial f}{\partial z}\cdot\vec{e}_3\right)$$
$$=\alpha\cdot\overrightarrow{grad}(f)+\beta\cdot\overrightarrow{grad}(g)$$

C.Q.F.D.